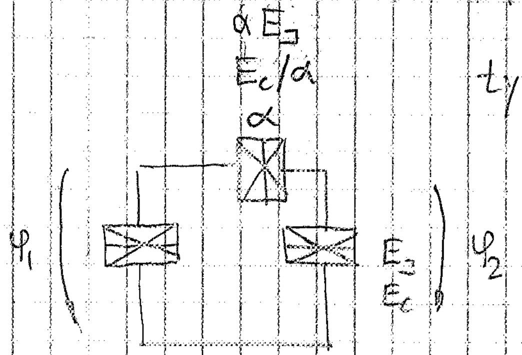
L10.1

## persistent-current qubit

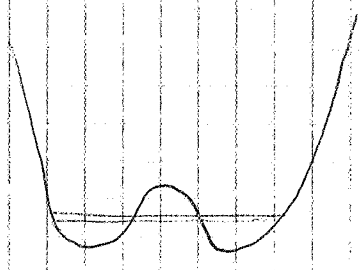



typical $\alpha = 0{,}8$

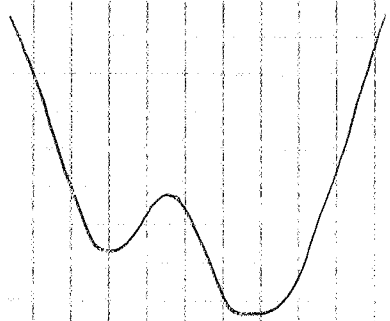
$$f = \frac{\Phi}{\Phi_0}$$

$$U = E_J\left[-\cos\varphi_1 - \cos\varphi_2 - \alpha\cos(2\pi f + \varphi_1 - \varphi_2)\right]$$

$f = 0{,}5$



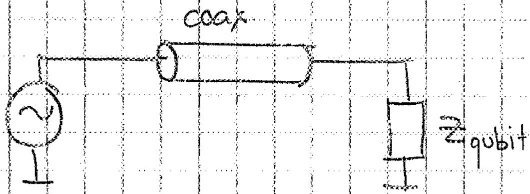
$f = 0{,}55$

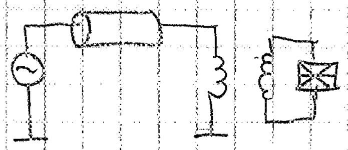


L10.2

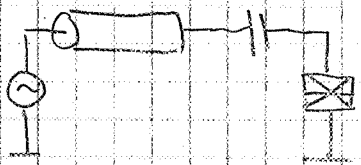
## Qubit relaxation / dephasing

coupling to environment

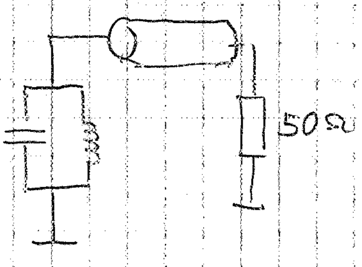


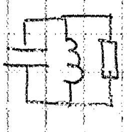



phase qubit



Cooper-pair box



RLC circuit

$$\omega = \frac{1}{\sqrt{LC}} \qquad \tau = \frac{1}{RC}$$

$\propto e^{-\frac{t}{\tau}}$

exponential decay

Fermi's golden rule

$$\Gamma = \frac{2\pi}{\hbar} |\langle f | H' | i \rangle|^2 \rho$$

L10.3

## noise & dissipation

random signal $V(t)$

auto correlation

$$G_{VV}(t-t') = \langle V(t)\, V(t') \rangle = \langle V(t-t')\, V(0) \rangle$$

assumed stationary noise

$$\tilde{V}(\omega) = \int e^{i\omega t}\, V(t)\, dt$$

power spectral density

$$S_{VV}(\omega) = \langle |\tilde{V}(\omega)|^2 \rangle$$

since $V(t)$ is real
$S_{VV}(-\omega) = S_{VV}(+\omega)$

Wiener - Kinchin theorem

$$S_{VV} = \int_{-\infty}^{\infty} dt\, e^{i\omega t}\, G_{VV}(t)$$

example: white noise

$$G_{VV}(t) = \sigma^2 \delta(t)$$

$$S_{VV}(\omega) = \sigma^2$$

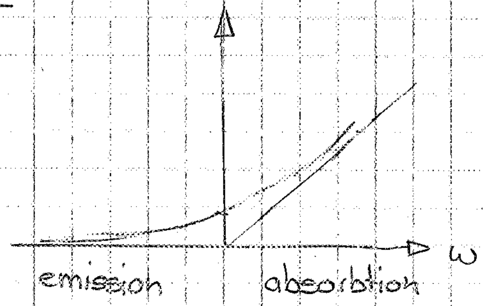
## quantum noise

$$S_{xx}(\omega) = \int_{-\infty}^{\infty} dt\, e^{i\omega t} \langle \hat{x}(t)\, \hat{x}(0) \rangle$$

since $\hat{x}$ is an operator
$S_{xx}(-\omega) \neq S_{xx}(+\omega)$

## quantum noise of a resistor

$$S_{VV}(\omega) = \frac{2R\,\hbar\omega}{1 - e^{-\hbar\omega/k_B T}}$$




for $k_B T \gg \hbar\omega$ ('classical limit')

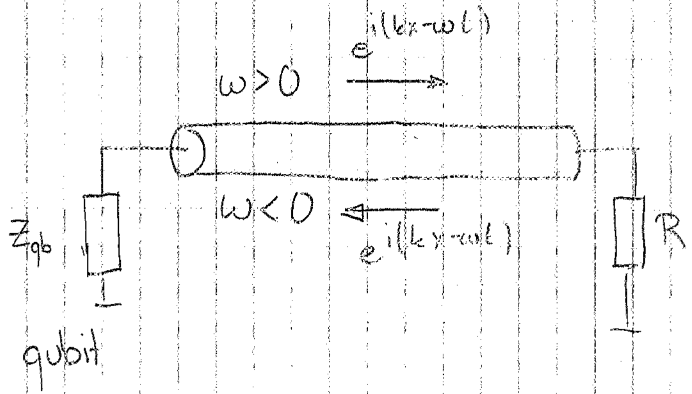
$$S_{VV}(\omega) = 2R\,k_B T$$

Johnson noise formula $S_{VV}(\omega) + S_{VV}(-\omega)$

'quantum limit'

Heavy-side function

$$S_{VV}(\omega) = 2R\,\hbar\omega\,\Theta(\omega)$$



## coupling to two-level system

$$\mathcal{H}_0 = \frac{\hbar\omega_{01}}{2}\hat{\sigma}_z$$

$$\mathcal{H}' = A\,\mathcal{F}(t)\,\hat{\sigma}_x$$

perturbation theory, first order (Fermi's golden rule)

$$\Gamma_\downarrow = \frac{A^2}{\hbar^2} S_{FF}(+\omega_{01})$$

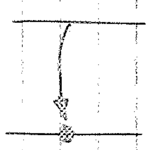
$$\Gamma_\uparrow = \frac{A^2}{\hbar} S_{FF}(-\omega_{01})$$

dephasing

$$\Gamma_\phi \propto S_{FF}(\omega \approx 0)$$

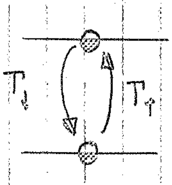
special form of the dissipation-fluctuation theorem

$k_B T \ll \hbar\omega_0$



qubit relaxes to ground state

$k_B T \gg \hbar\omega_0$




$\Gamma_\downarrow \approx \Gamma_\uparrow$

qubit is in an equal mixture of ground and excited state